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Analysis and Mathematical Application of Taylor Series to Crystallization of c minerals from Magma

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Abstract

Power series in magma is the power of a mineral to form from magma at a control condition of temperature or pressure and the process of mineral formation from magma using power series is called Taylor series in Magma. The reactions in Magma is very complex, because of several anionic and cationic substitutions of different size and charge in the magma and in this case, complex problems require complex solutions. This research was focused more on using Bowen and Goldschmidt concepts concerning the elemental substitution and distribution of chemical elements in rocks throughout the time of crystallization with mathematical foundation such as Taylor series to predict major minerals encompass the olivine, pyroxene, amphibole, mica, and feldspar, and their associated rocks encompass granite, basalt, andesite, and trachyte. Findings have shown that, in mathematical context, Bowen's and Goldschmidt rules were mathematically connected using the Taylor series as;

Keywords: Mathematical, Application, Taylor, Series

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Introduction

The application of mathematical concepts, such as power series, in engineering and science is commonplace. These series have been used to solve a wide range of problems in machine learning, mechanics, electricity, physics, and chemistry, as noted by (Zekić, 2025). However, their

application to geology, mineralogy, and petrology remains less explored. This text proposes that the sequential crystallization of minerals from cooling magma, a thermodynamically driven process, can be described mathematically using power series. This mathematical framework can provide a formal representation of the Bowen reaction series, a fundamental principle in petrology. The Bowen reaction series, first proposed by Norman L. Bowen in 1928, explains how minerals form from cooling magma. As the temperature of the magma decreases, minerals crystallize in a specific and predictable sequence. This process encompasses two main series: the discontinuous series (where minerals react with residual melt to form new minerals with different crystal structures) and the continuous series (where minerals of the same group, such as plagioclase feldspar, gradually change their chemical composition by exchanging elements with the melt). Each mineral crystallizes under specific thermodynamic conditions, such as a specific temperature or pressure.

This crystallization process can be conceptualized as a mathematical series. We can define a "magma power series" as the potential for a mineral to form from magma under a specific set of thermodynamic conditions (e.g., discrete temperatures or pressures). This differs from the idea that minerals of the "same power" will have different temperatures and pressures. Instead, a continuous decrease in temperature leads to sequential crystallization of minerals, with each mineral forming at a specific temperature range until the melt is exhausted. The processes described by Bowen's reaction series, particularly element exchange and substitution, are governed by geochemical principles (Worlitschek & Mazzotti, 2004). The classic Goldschmidt rule (1937) in (Akimov et al., 2016) explains how elements are distributed during crystallization. This rule states that: Ions of similar size and charge can substitute for each other in the crystal lattice; Ions with smaller ionic radii (ideally less than 15%) can substitute for each other more easily; Ions with higher ionic potentials (charge-to-radius ratios) will form stronger bonds and preferentially enter the crystal lattice. Later, (Kawabe, 1977) proposed a modification by adding the concept of electronegativity to better understand the distribution of chemical elements, especially in minerals with a high percentage of covalent bonds. Bernard J. (Osborn, 1950) further refined Goldschmidt's rule, emphasizing that for ions of the same charge, the ion with the ionic radius closest to the radius of the preferred site will enter the lattice most readily (Zhang et al., 2019).

In the mathematical analysis of magma crystallization using power series, we can assume $Z0$ represents the nucleation point of the first mineral to form—the building block for all subsequent silicate minerals (Shorttle et al., 2016). Let Z denote any other nucleation point formed at a given crystallization time. The function $f(Z)$ represents the magma state, which is assumed to be differentiated η times during crystallization, and can be constrained by the conditions within the magma chamber, from the initial nucleation point $Z0$ to the end of crystallization ϖ. This can be written as $Z0 \leq Z \leq \varpi$. Taylor series can be used to model changes in the system as a function of temperature, pressure, or composition. For such a model to work, the function $f(Z)$ must be differentiable in the open interval $Z0 < Z < \varpi$ and continuous in the closed interval $Z0 \leq Z \leq \varpi$. If a point ω exists such that $Z0 < \omega < \varpi$, then the Taylor series can mathematically describe the sequential nature of the Bowen reaction series. This repeated sequence of crystallization and separation of minerals from the melt is known as fractional crystallization. This process can be viewed as the convergence of a power series, where each “fraction” or domain of convergence represents a new stage of magma evolution where new crystal nucleation occurs (Tonn et al., 2024).

This approach builds on existing mathematical methods used in other fields. For example, the Adams-Bashforth method, a numerical technique for solving ordinary differential equations, was developed by John Couch Adams to model capillary action in collaboration with Francis (Yi et al., 2023). Recent research, such as that by (Reeder & Rakovan, 1999), has applied this method to problems in physics and engineering, such as population growth. While not directly related to power series, this demonstrates the successful application of complex numerical methods to

physical and chemical processes, providing a conceptual precedent for applying sophisticated mathematical tools to petrology.

Methodology

Power series in magma is the sequence of crystallization of magma into different minerals of various degrees of polynomials by Taylor and Euler series with respect to thermodynamics. Given that;

$$\boldsymbol{f^{1}(ZZ_{0})o} = \boldsymbol{f^{2}(ZZ_{0})}\text{o}, \boldsymbol{\eta} = 2$$
$$\boldsymbol{f^{2}(ZZ_{0})o} = \boldsymbol{f^{1}(ZZ_{0})p}, \boldsymbol{\eta} = 1$$
$$\boldsymbol{f^{1}(ZZ_{0})a} = \boldsymbol{f^{7}(ZZ_{0})}\text{a}, \boldsymbol{\eta} = 7$$
$$\boldsymbol{f^{2}(ZZ_{0})a} = \boldsymbol{f^{6}(ZZ_{0})}\text{m}, \boldsymbol{\eta} = 6$$
$$f^{1}(ZZ_{0})o = f^{6}(ZZ_{0})\text{m}, \boldsymbol{\eta} = 6$$
$$\boldsymbol{f^{2}(ZZ_{0})o} = \boldsymbol{f^{5}(ZZ_{0})}\text{m}, \boldsymbol{\eta} = 5$$

$$f^{\eta}(ZZ_{0})\boldsymbol{\Psi} = \boldsymbol{f^{1}(ZZ_{0})} + f^{11}(ZZ_{0}) + f^{111}(ZZ_{0}) + f^{4}(ZZ_{0}) + \boldsymbol{f^{5}(ZZ_{0})} + \boldsymbol{f^{6}(ZZ_{0})} + f^{8}(ZZ_{0}).$$

$$f^{\eta}(ZZ_{0})\boldsymbol{\Psi} = \boldsymbol{f^{1}(ZZ_{0})p} + \boldsymbol{f^{11}(ZZ_{0})o} + f^{111}(ZZ_{0}) + f^{4}(ZZ_{0}) + f^{5}(ZZ_{0}) + \boldsymbol{f^{6}(ZZ_{0})m} + \boldsymbol{f^{7}(ZZ_{0})a} + f^{8}(ZZ_{0}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$\boldsymbol{\Psi}$ = Olivine (o) +Pyroxene (p) + Amphibole (a) + Mica (m).

Therefore;
Power series in Magma $\mathfrak{V}$, *involves the Following Series*

- Euler's series
- Taylor's Series
- Maclaurine's Series

Euler's, Taylor's, and Maclaurine's series are applied to resolve the problems associated with chemical distribution and elemental substitution in Rocks during crystallization of magma. In order to resolve these problems, silicate identity must be established, for each silicate mineral, during crystallization, using Euler's series.

1: EULER SERIES

Given that;

$$ex = 1 + x + x2 + x3 + \cdots xn + \cdots 1 \ldots\ldots\ldots (4)$$

Given that; 2! 3! $n!$ $f(\text{x}) = \text{e}x\text{x} = Z0$

$ex \longrightarrow eZ0$ in the horizontal axis of the complex plane of magma $\mathfrak{V}$, then For complex $Z0$;

$$f(Z0) = eZ0 \ldots\ldots\ldots (5)$$

$f(Z0) = eZ0$ is the unique differentiable function of a fixed complex of which the derivative equals the function in the magma, $\mathfrak{W}$

$$ex = eZ0$$
$$eZ0 = 1 + Z0 + (Z0)2 + 2! (Z0)33! + . . + (Z0)nn!$$

If $Z0$ is a fixed complex in silicate magma, then Z and ω are complex variables in silicate

For $Z0 = 0$, then;
$f(Z) = f0(Z0) + f1(Z0)\ Z + f11\ (Z0)(Z\ )2. . +. .\ f\eta\ (Z0)(Z\ )\eta{-}p$ (13)
$f(Z) = f0(0) + f1(0)Z + f11(0)(Z\ )2. . +. .\ f\eta(0)(Z\ )\eta{-}p$ (14)

Therefore equation (14) above is called Maclaurin's series Given that; Z = $Z - Z0$,Where $Z0$, is a fixed complex and Z is a complex variable in Magma then; $f(Z) = f\ 0(Z0) + f1(Z0)\ Z - Z0 + f11$ $(Z0)(Z - Z0)2. . +. .\ f\eta(Z0)(Z - Z0)\eta{-}p$ .. (15) Therefore equation (15) above is called Maclaurin's series in Magma $\mathfrak{V}$.

TAYLOR SERIES

Let a, be a fixed point in some interval and let x denote any other point in that.
If $f(x)$ is differentiable n times in the interval, a ≤ x ≤ b and if fn(x) is differentiable in, a < x< b and continuous in a ≤ x ≤ b, there exits ∈ in the interval, a < ∈ < b such that;
$f(b) = f0$(a) + $f1$(a) $(b - a) + f\ 11$ (a) $(b-$ a)2 + ⋯ + $f\eta$ (a) $(b-$ a) η +…. + $f\eta$+1 (a) $(b-$ a) η+1
2! η! $(\eta$+1)! $f(x) = f0$(a) + $f1$(a)$(x$ − a) + $f11$ (a)$(x-$ a)2 + ⋯ + $f\eta$ (a)$(x-$ a)η +…. R (x)
2! η! n, Where, Rn denotes the remainder term. Suppose now that;
a. $f(x)$ is infinitely differentiable over the interval concerned
b. For each x in the interval, we $LimRn$(x)= 0a → ∞
Then allowing n to tend to infinity in Taylor's theorem and obtain an infinite series expansion for $f(x)$, called Taylor series
$f(x)$, about the point x= a$f(x) = f0$(a) + $f1$(a)$(x$ − a) + $f11$ (a)$(x-$ a)2 + ⋯ + $f\eta$ (a)$(x-$ a)η +….
2! η ! In a particular case when a = 0, we get the Maclaurian series for $f(x)$. $f(x) = f0(0) + f1(0)$ $(x) + f11\ (0)\ (x\ )2 + \cdots + f\eta\ (0)$(x) η +….2! η!

Analytic Function

A function $\boldsymbol{f}$ of the complex variable Z is analytic at a point Z 0, if its derivative exists not only at $Z0$ but also at each point Z in some neighborhood of $Z0$. A function $\boldsymbol{f}$ is said to be analytic in a given region R if is analytic at each point in R. An entire function is a function that is analytic at each point in the entire finite plane. Since the derivative of a polynomial exists everywhere, it follows that every polynomial is an entire function.

Interval of Convergence and Radius of Convergence

For any given power series, $\infty\ \boldsymbol{n}\ \sum$ ( ) n=1 $\boldsymbol{anx}$}, there will be just three possibilities;

i. The series converges when x = 0 and diverges for every value of x.
ii. The series converges for every value of x
iii. There exists a finite positive number R, such that the series converges absolutely for every value of x such that |x| < R and diverges for every other value of x such that |x| > R.

Analytic Function in Magma, $\mathfrak{W}$ using Taylor series.

Let $Z0$ be a nucleation point and represents building block of all silicate minerals during the time of crystallization and with respect to thermodynamic conditions, let Z denote any other formed nuclei point at that time of crystallization.
If $f(Z)$ is a differentiable n times during the time of crystallization, $Z0 \le Z \le \mathfrak{W}$ and if fn(Z) is differentiable in, $\boldsymbol{Z0} < Z < \mathfrak{W}$ and continuous in $Z0 \le Z \le \mathfrak{W}$, there exits ω during the

time of crystallization that is, $\boldsymbol{Z0} < \omega < \mathfrak{W}$, such that for Taylor series in magma we have that; 112$\eta\eta f(Z)$ = $f0(Z)$ + $f1(Z)$ $(Z - Z)$ + f $(Z0)$ $(Z- Z0)$ + ⋯ + f $(Z0)$ $(Z- Z0)$ +⋯ (16) 002! η! Then; $f(Z)$ = (Z) + $(x + iy)$ (Z) + $(x+iy)2$ (Z) + $(x+iy)3$ (Z) + ⋯ $(x+iy)$ η (Z) …… (17)002!0 3!0η!0$f(Z)$ = ω, Where; $Z0\ldots$ $Z0\ldots$ $Z0\eta$ = Silicate identityη! f (ZZ) = $(x+iy)$ η $Z0\eta0\eta$! η ! f $(Z0)$ = $eZ0$(Euler equation) $f2(ZZ0\eta(x+iy)$ $\eta-p$ $Z\eta\eta$! = ($\boldsymbol{\eta}$-p) $\boldsymbol{\eta}$ = 0 ⟶ $\infty f2(ZZ0f2(ZZ\eta(x+iy)$ $\eta-p$ $Z\eta(\eta-p))$ $((x+iy)$ $\eta-p)$ eZ…. (18)00$(\eta-p)$

Results and Discussion

Bowen explanation using taylor series in magma $\mathfrak{W}$

By definition, Bowen's reaction series with respect to Taylor series in Magma states that, whenever there is continuous drop in temperature, minerals tend to crystallize sequentially according to Bowen (1928) and then generate sequence of crystallization which fractionates into differential fractions called Magmatic differentiation and the entire crystallization process in which early formed crystals are removed from magma (Yuguchi et al., 2020); (Kruger & Latypov, 2022), enriching the remaining liquid in other elements is called fractional crystallization as defined in the Taylor Magma series below.

$\boldsymbol{f(ZZ)\Psi = f0(Z) + f1(Z)(Z - Z) + f\ (Z0)(Z- Z0) + \cdots + f\ (Z0)(Z- Z0) + \cdots + \ldots . 00002!\eta!\Phi\eta(ZZ0)}$ …… (19)

- $\boldsymbol{f0(Z0)}$: nucleation point in the magma $\mathfrak{W}0$, such that;

f ($Z0$) is differentiable at point $Z0$

- $\boldsymbol{f1(Z0)(Z - Z0)}$: first fraction in the magma $\mathfrak{W}1$, such that;

f (Z) is differentiable at point $f1(Z0)$

- $\boldsymbol{f11\ (Z0)(Z- Z0)2}$: second fraction in the magma $\mathfrak{W}$, such that;$\boldsymbol{2!}$ 2

f (Z) is differentiable at point $f11(Z0)$

- $\boldsymbol{f\eta\ (Z0)(Z- Z0)\eta}$: nth fraction in the magma $\mathfrak{W}$, such that;$\boldsymbol{\eta!}$ n

f (Z) is differentiable at point $f\eta(Z0)$.

- $\boldsymbol{\Phi\eta}$(Z): is the remaining fluid in the magma $\mathfrak{W}$, such that;

Φ (Z) is differentiable at point $f\eta(Z0)$.

Where, $\boldsymbol{\Phi\eta}$ denotes the remainder term. Suppose now that, $f(Z)$ is infinitely differentiable over the time of crystallization, then, for each Z in the time of crystallization, we have;$\boldsymbol{Lim\ \Phi\eta(Z) = 0Z0 \rightarrow \infty}$ Then allowing $\boldsymbol{\eta}$ to tend to infinity in Taylor's theorem and obtain an infinite series expansion for $f(Z)$, called Taylor series in Magma, $\mathfrak{W}$.

According to Bowen's explanation as shown in Table (1) and Figure (1) below, minerals progress to the next minerals through polymerization reaction by addition of more silica to the already crystallized minerals in a more silica rich melt. In Geology, magmatic differentiation is an umbrella term for the various processes by which magmas undergo bulk chemical change during the partial melting and cooling. The sequence of magmas produced by igneous differentiation is known as Magma series. This means that,

1. when considering cooling of magma, with continuous drop in temperature of magma, olivine with the content of $(\boldsymbol{f11}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{2})$o will be integrated into pyroxene with the$\boldsymbol{2}$! content of $(\boldsymbol{f111}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{3})$p, pyroxene will be integrated into imphibole with the content$\boldsymbol{3}$!of $(\boldsymbol{fvii}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{7})\boldsymbol{a}$, and mica with the content of $(fvi\ (Z0)(Z-Z0)6)m$, and the remainder $\boldsymbol{2}$! 2!
with the content of $[\Phi n(\boldsymbol{Z0})]$l will be quenched at infinity to produce the last crystals such as orthoclase, muscovite and quart as, $\boldsymbol{Lim\ \Phi\eta(Z) = 0Z0 \rightarrow \infty}$In the contrary,

2. why during melting with continuous increase in temperature of Rock, Amphibole with the content of $(\boldsymbol{f7}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{7})$a, will be differentiated into more stable Mica with the$\boldsymbol{7}$! content of $(\boldsymbol{f6}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{6})\boldsymbol{m}$ and Pyroxene with the content of $(\boldsymbol{f111}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{3}),\boldsymbol{6}!$ $\boldsymbol{3}$!pyroxene will be differentiated into Olivine with the content of $(\boldsymbol{fii}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{2})\boldsymbol{a}$, and$\boldsymbol{2}$! then the entire rock with the content of $[\boldsymbol{\Psi}n(\boldsymbol{ZZ0})]$ will be melted and differentiated to infinity to produce one homogeneous magma as, $\boldsymbol{Lim\ \mathfrak{B}\eta(Z) = \infty\ Z0 \rightarrow 0}$

This magmatic differentiation in Magma called Taylor series in Magma because its application requires Calculus which are the differentiation and integration in magma under thermodynamic change such that,$\boldsymbol{11\ \ 2\ \ \ vii\ \ \ 7f(ZZ\ )\Psi} = \boldsymbol{f0(Z\ )} + \{\boldsymbol{f1(Z\ )(\ Z-Z\ )}\}\boldsymbol{p} + \{\boldsymbol{f}$ $(\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\ \}\ \boldsymbol{o} + \{\boldsymbol{f}(\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\ \}\ \boldsymbol{a} + \boldsymbol{00002!7!}\{\ \boldsymbol{fvi}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{6}\}\ \boldsymbol{m} + \cdots + \{\boldsymbol{f\eta}$ $(\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{\eta}\}\boldsymbol{nth} + \cdots + \ldots.\ \{\boldsymbol{\Phi}\ (\boldsymbol{Z}\ )\}$l ……. (20)
$\boldsymbol{6!\eta!n\ \ 0f0(Z0)}$ = Silicate identity

- $\boldsymbol{f1(Z0)(\ Z-Z0)}$ = Pyroxene(p) composition

- $\boldsymbol{f11\ (Z0)(Z-Z0)2}$ = Olivine(o) composition$\boldsymbol{2}$!

- $\{\boldsymbol{fvii}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{7}\}\ \boldsymbol{a}$ = Amphibole(a) composition$\boldsymbol{7}$!

- $\{\boldsymbol{fvi}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{6}\}\ \boldsymbol{m}$ = Mica (m) composition$\boldsymbol{6}$!

- $\boldsymbol{f\eta}\ (\boldsymbol{Z0})(\boldsymbol{Z}-\boldsymbol{Z0})\boldsymbol{\eta}$ = nth mineral$\boldsymbol{\eta}$!

- $\{\boldsymbol{\Phi n}(\boldsymbol{Z0})\}$l = Felsic composition (orthoclase, Muscovite and Quartz)

3. During chemical reactions in magma with respect to Taylor series, with the concept of thermodynamic conditions, the following sequence of reactions below are used to explain the crystallization of minerals and the evolution of igneous rocks from the magma at different temperatures with respect to Taylor series in magma.

- Magma at its highest potential energy and high temperature consists of mostly abundant of elements of silicon and oxygen which is represented in equation (21) below.

$Si + O2 \rightarrow SiO2$ …….. (21)

- As the magma gradually loses its potential energy and becomes more kinetic energy, Magma becomes more viscous in which silica in the magma reacts with two atoms of oxygen to produce discrete silicate tetrahedral unit as shown in equation (22), below.

$SiO2 + O2 \rightarrow (SiO4)-\boldsymbol{4}$ (22) $(SiO4)-\boldsymbol{4}$ = silicate independent tetrahedral unit :

- The gain in kinetic energy with drop in temperature of the magma would cause the discrete silicate unit formed from equation (22) to react with the metallic oxide (MgO) in the magma and

with certain potential energy, produces two polymorphs of the silicate minerals, pyroxene (**Mg $SiO3$**) and olivine (**Mg2$SiO4$**) as shown in equations (23) and (24), below.

 The early Pyroxene formed in equation (23) below, is found to be existed in the magma even before cooling begins. It was found to be green crystal in many lava and according to (Schwinger & Breuer, 2022) its occurrence in the lava at that temperature is accidental, but the reaction that results its formation in the lava is represented in equation (23), above.

MgO + $SiO2$ → **Mg $SiO3$** (23)

($SiO3$)−**2** = Cyclic silicate

Mg $SiO3$ = Pyroxene

 When the lava gains kinetic energy with decrease in temperature and cools further to reach a maximum potential energy (that is from kinetic energy to potential), a more stable crystal of olivine of discrete tetrahedral unit is formed from the melt according to Bowen (1928) in (Marsh, 2013)as shown in equation (24), below. Which means that the Olivine formed at that temperature and energy does not link its ($SiO4$)−**4** tetrahedral unit to other tetrahedral units by sharing oxygen atoms at the corner of each tetrahedral unit, which makes Olivine to be independent tetrahedral unit and this type of silicate of independent tetrahedral unit is called Nesosilicate.

Mg $SiO3$ + MgO → **Mg2$SiO4$** (24)

$SiO4$ = Olivine identity,

Mg2$SiO4$ = Olivine

 When the cooling continues sequentially, early olivine formed, polymerized into more stable pyroxene by the addition of more silica to the crystal of olivine as shown in Figure (1), below. Olivine with the chemical formula of (**Mg2$Si\ O4$**)formed from equation (24) above, undergoes polymerization reaction by addition of silica (SiO2) by linking ($SiO4$)−**4** tetrahedral units linearly, with each unit shares two oxygen atoms with other units to form a chain pyroxene crystal and the chemical reaction is represented in equation (25), below.

Mg2$SiO4$ + $SiO2$ → **Mg2$Si2O6$** ⋯⋯⋯ ⋯⋯. (25)

Mg2$Si2O6$ = **Mg2**(**$SiO3$**)**2**

$SiO3$ = Pyroxene identity

Mg2(**$SiO3$**)**2** = Pyroxene

 The reaction in magma continues with sequential drop in temperature such that the earlier chain silicate formed from equation (25) above reacts with the metallic oxide in the magma to produce a Pyrosilicate in which two **$SiO4$** tetrahedral units are linked with each other by sharing one oxygen atom at the corner of the tetrahedral units and the chemical reaction is represented in equation (26), below. Examples of a mineral with this type of pyro silicate structure is thortveitite with chemical formula of **Sc2$Si2O7$** and is called Sorosilicate.

Mg2$Si2O6$ + MgO → **Mg3$Si2O7$** (26)

 as magma continues to cool, and evolves more minerals from the melt, the pyro silicate (**Mg3*Si2O7***)formed from equation (26) above, undergoes polymerization reaction by addition of silica (SiO2) to the existing crystal to form a cyclic silicate crystal with chemical formula of **Mg3*Si3O9***, which are linked by three or more tetrahedral units cyclically, by sharing two oxygen atoms with adjacent tetrahedral units and this is represented in equation (27), below. Examples of the minerals with this type of cyclic structure are Beryl(**Be3*Al2Si6O18***) or tourmaline and Benitoite (**BaTi*Si3O9***) and is called Ring structure.

Mg3*Si2O7* + SiO2 → **Mg3*Si3O9*** (27)
Mg3*Si3O9* ⟶ **Mg3(*SiO3*)3**
SiO3 = Pyroxene identity in rocks
Ca3(*SiO3*)3 = Wollastonite1

 More minerals continue to evolve from the melt as magma continues its cooling, and therefore, the pyro silicate mineral (**Mg3*Si3O9***), formed from equation (27) above, react with pyroxene to form a double chain pyroxene crystal where two chains are linked together so that each tetrahedral group shares three of its oxygen atoms with chemical formula of **Mg4*Si4O12***, and is represented in equation (28), below. Examples of minerals with this type of pyro silicate structure are Amphibole (**Mg3.5*Si4O11OH***)and Mica
{**Mg3*Si4O10*(*OH*)2**}and are called Innosilicate.
Mg3*Si*3*O*9 + Mg***SiO3*** → **Mg4*Si4O12*** (28)

 If the magma is more hydrous and viscous than the original melt 𝔚, then equation (28) above, becomes equations (29) and (30), below.
Therefore given that;

H2O → **OH**− + **H**+
Then;
Mg4*Si4O*12 + OH → **Mg3.5*Si4O11*(OH)** (29)
Mg4*Si4O*12 + 2(OH) → **Mg3*Si4O10*(*OH*)2**⋯. (30)

 The remaining Magma cools to infinity, to form felsic minerals in which silicate tetrahedral (***SiO4***) unit links all its corner with another tetrahedral units by sharing four oxygen atoms to form a framework structure. Examples of minerals with this type of frame structure structure are Orthoclase (**K1*AlSi3O8***)and Quartz {***SiO2***}and are called Tectosilicate.

4. For the sequential order of crystallization with respect to Taylor series in the magma, then we have that;

f*(*ZZ*0)Ψ** = **Mg*SiO3 + **Mg4*SiO4*** + **Mg3*Si3O9*** + **Mg4*Si4O12*** + **K1*AlSi3O8*** + ***SiO2***.. (31)
***f*(*ZZ*0)Ψ** = Enstatite + Forsterite + Enstatie + Kupfferite + orthoclase + quartz
f*(*Z*)** = ***SiO + ***SiO*** + (***Si O4***) + (***Si3 O9***) + (***Si4 O12***) +…… … + (***SiO2***) ….. (32)

0232!**3**!4! ∞! ***f*(*ZZ*0)Ψ** = ***SiO2***+ ***MgSiO***

+ (***Mg2Si O4***) + (***Mg3Si3O9***) + (***Mg4Si4 12***) +… + (**K**,**Al**,***SiO2***) ..(33)

2! 3! 4! ∞!$\boldsymbol{f(ZZ)\Psi = SiO + MgSiO + (Mg2Si\,O4) + (Mg3Si3O9) + \{Mg7Si8O22\,(OH)2\}}$ +. . + (**K**,**Al**,$\boldsymbol{SiO2}$) ..(34)

$\boldsymbol{0232!3!}$7!∞!$\boldsymbol{f(ZZ)\Psi = SiO + MgSiO + (Mg2Si\,O4) + (Mg3Si3O9) + \{Mg6Si8O20(OH)4\}}$ +. . + (**K**,**Al**,$\boldsymbol{SiO2}$) ..(35)

$\boldsymbol{0232!3!}$6!∞!$\boldsymbol{f(ZZ)\Psi}$ = **Silica** + (**Pyroxene**) + (**Olivin**e) + (**Pyroxene**) + (**Amphibole**) +. . + (**Felsic**) … (36)

02!3!4!∞!$\boldsymbol{f(ZZ)\Psi}$ = **Silica** + (**Pyroxene**) + (**Olivine**) +(**Pyroxene**) +(**Mica**) +. . + (**Felsic**) ….(37)

Goldschmidt explanation using taylor series in magma

By definition of Goldschmidt substitution rules (Goldschmidt, 1926) in (Eskandari & Sadeghi, 2024) using Taylor series in Magma, which states that, ions substitute for each other in the crystal lattice of a mineral if there is Diadochi in the crystal and the expressions are represented in equations (38), (39), (40), (41),
(42), (43) below. Given that.

- $f0(Z0) = Z, Z = \beta + i\alpha$.

Then Z forms a Diadochi in the magma as represented below.

- $f1(ZZ0) = (\beta + i\alpha)(Z0)$
- $f11(ZZ) = (\beta+i\alpha)2\,(Z)$

0 2! 0

- $f111(ZZ) = (\beta+i\alpha)3\,(Z)$

0 3! 0

- $f\eta(ZZ) = (\beta+i\alpha)\eta\,(Z)$

0 η! 0

Given that,

β = Mg
α = Fe and Z = **β** + α
Then for sequence of crystallization using Taylor series, we have that;

$f\,(ZZ) = (Z) + (\beta + i\alpha)(Z) + (\beta+i\alpha)2\,(Z) + (\beta+i\alpha)3\,(Z) + \cdots (\beta+i\alpha)\eta\,(Z)$ …… (38)

0002!03!0η!0$f(ZZ0)$ = ω
Where;
$Z0\ldots\ Z0\ldots\ldots Z0\eta$ = Silicate identity
$\eta!\boldsymbol{f(ZZ0)\Psi}$ = **MgFe**$\boldsymbol{SiO3}$ + (**MgFe**)**4**$\boldsymbol{SiO4}$ + (**MgFe**)**3**$\boldsymbol{Si3O9}$ + (**MgFe**)**4**$\boldsymbol{Si4O12}$

+(**KNa**)**1**$\boldsymbol{AlSi3O8}$ + $\boldsymbol{SiO2}$.. (39)
$\boldsymbol{f(ZZ0)\Psi}$ = Pyroxene + Olivine + Pyroxene + Amphibole + orthoclase + quartz $\boldsymbol{f(Z)}$ = $\boldsymbol{SiO}$+ $\boldsymbol{SiO}$+ ($\boldsymbol{Si\,O4}$) + ($\boldsymbol{Si3\,O9}$) + ($\boldsymbol{Si4\,O12}$) +…… … + ($\boldsymbol{SiO2}$) ….. (40)

0232!**3**! 4! ∞!$\boldsymbol{f(ZZ)\Psi}$ = $\boldsymbol{SiO}$+ ($\boldsymbol{Mg}$, $\boldsymbol{Fe}$)$\boldsymbol{SiO}$+ ($\boldsymbol{Mg}$,$\boldsymbol{Fe}$)**2**$\boldsymbol{Si\ O4}$)
+((**Mg**,**Fe**)**3**$\boldsymbol{Si3O9}$) + {(**Mg**,**Fe**)**4**$\boldsymbol{Si4\ 12}$)} +… +**0 2**(**K**,**Al**,$\boldsymbol{SiO2}$) ..(41)
∞!
32!3!4! $\boldsymbol{f\ (ZZ)\ \Psi}$ = $\boldsymbol{SiO}$+ ($\boldsymbol{Mg}$, $\boldsymbol{Fe}$)$\boldsymbol{SiO}$+ {($\boldsymbol{Mg}$,$\boldsymbol{Fe}$)**2**$\boldsymbol{Si\ O4}$} + ((**Mg**,**Fe**)**3**$\boldsymbol{Si3O9}$) +

0232!**3**!{(**Mg**,**Fe**)**7**$\boldsymbol{Si8O22}$ ($\boldsymbol{OH}$)**2**} +. . + (**K**,,**Al**,,$\boldsymbol{SiO2}$) (42)

$\boldsymbol{f(ZZ}$7!) $\boldsymbol{\Psi}$ = $\boldsymbol{SiO}$∞!+ ($\boldsymbol{Mg}$, $\boldsymbol{Fe}$)$\boldsymbol{SiO}$+((**Mg**,**Fe**)**2**$\boldsymbol{Si\ O4}$) + {($\boldsymbol{Mg}$,$\boldsymbol{Fe}$)**3**$\boldsymbol{Si3O9}$) +

0232!**3**!{{**Mg**,**Fe**)**6**$\boldsymbol{Si8O20(OH)4}$ } +. . + (**K**,**Al**,$\boldsymbol{SiO2}$) ..(43)
6! ∞!$\boldsymbol{f(ZZ)\Psi}$ = **Silica** + (**Pyroxene**) + (**Olivin**e) + (**Pyroxene**) + (**Amphibole**) +. . + (**Felsic**) … (44)

02!3!4!∞!$\boldsymbol{f(ZZ)\Psi}$ = **Silica** + (**Pyroxene**) + (**Olivine**) +(**Pyroxene**) +(**Mica**) +. . + (**Felsic**) ……. (45)

02!3!4!∞!

Bowen's considers the reaction series of plagioclase feldspar as Continuous reaction series because of its continuous exchange of element for one another with drop in temperature (Haldar & Tišljar, 2014). Therefore with mathematical foundation in this research, this reaction series is considered to be Isomorphous reaction series because, it requires elements of similar charge and size to substitute for one another particularly if there is Diadochi in the crystal of mineral in which the substitution is taking place as shown in Table (1) below, such that first Plagioclase feldspar from the melt is enriched in calcium and poor in sodium under thermodynamic change. Therefore for mafic minerals, this first mafic mineral to crystallize from the melt is forsterite of Olivine and the 'Forsterite 'under thermodynamic change, would undergo ionic substitution to form fayalite as the end member of olivine in the olivine series (Oh et al., 2023). Both forsterite, wholly magnesium rich olivine and fayalite, wholly iron rich are formed within the olivine series as shown in Tables 1, 2, Figures 1 and 2. The second mafic mineral to crystallize from the melt is Enstatite of Pyroxene and the 'Enstatite 'under thermodynamic change, would undergo ionic substitution to form ferrosilite of pyroxene as the end member of pyroxene in the pyroxene series (Wang et al., 2021). Both Enstatite, wholly magnesium rich pyroxene and Ferrosilite, wholly iron rich are formed within the pyroxene series as shown in Tables 1, 2, Figures, 1 and 2 below.

The third mafic mineral to crystallize from the melt is Kupfferite of Amphibole and the 'Kupfferite 'under thermodynamic change, would undergo ionic substitution to form Grunerite of amphibole as the end member of amphibole in the amphibole series (Yang et al., 2022). Both Kupfferite, wholly magnesium rich amphibole and Grunerite, wholly iron rich amphibole are formed within the amphibole series as shown in Tables 1, 2, Figures, 1 and 2 below. The fourth mafic mineral to crystallize from the melt is Phlogopite of Mica and the 'Phlogopite 'under thermodynamic change, would undergo ionic substitution to form Lepidomelane of mica as the end member of mica in the mica series (Korneev et al., 2022). Both Phlogopite, wholly magnesium rich mica and Lepidomelane, wholly iron rich mica are formed within the mica series as shown in Tables 1, 2, Figures, 1 and 2 below. Number 5 in the series according to Bowen's is Orthoclase, a potassium rich alkali feldspar. Sometimes sodium can substitute for potassium in orthoclase to form the end member, albite, a sodium rich alkali feldspar in the alkali series as shown in Table 2, Figures 3, 4 and 5 bellow. Both the potassium rich orthoclase and sodium rich Albite form alkali series and they exist at high temperature as Perthite – Antiperthite mix ed

feldspar and have separate phase at lower temperature as perthite when orthoclase exceds albite and Antiperthite when albite exceeds orthoclase.

This process is called EXSOLUTION in rock forming minerals from the alkali melt αm. The sixth mineral to crystallize from the melt is Muscovite of white Mica and the 'Muscovite 'under thermodynamic change, would undergo ionic substitution to form Paragonite of white mica as the end member of white mica in the white mica series. Both Muscovite, wholly potassium rich white mica and Paragonite, wholly sodium rich white mica are formed within the white mica series as shown in Tables 1, 2, Figures, 1 and 2 below. The process in which minerals formed by substitution of one element for anther using Goldschmidt concept is called Isomorphous series.

Table 1. Order of Crystallization of Minerals using Goldschmidt Concept.

Tem (^{0}C)	Feldspar	Number, N	MAFIC						
			1	2	3	4	5	6	7
		FELSIC.	OLIVINE	PYROXENE	AMPHIBOLE	BLACK MICA	FELDSPAR	WHITE MICA	SILICA
2000	Anorthite	1	Forsterite	X	X	X	X	x	x
1000	Bytownite	2	Mgnesio-chrysolite	Enstatite	X	X	X	x	x
900	Labradorite	3	Chysolite	Mnesio-hypersthene	Kupfferite	X	X	x	x
800	Andesine	4	Hyalosiderite	Hypersthene	Magnesio-Anthophyllite	Phlogopite	X	x	x
700	Oligoclase	5	Hortonolite	Ferro-hypersthene	Anthophyllite	Biotite	Orthoclase	x	x
600	Albite	6	Ferr–hortonolite	Eulite	Cummingtonite	Ferro-biotite	Perthite	Muscovite	x
500	Quartz	7	Fayalite	Ferrosilite	Grunerite	Lepidomelane	Albite	Paragonite	Quartz

Table 2. Order of Crystallization of Minerals using Bowens and Goldschmidt Concepts.

Tem (^{0}C)		Number (η)							
			1	2	3	4	5	6	7
	Order of Crystallization	FELSIC SERIES.	MAFIC SERIES						
		PLAGIOCLASE FELDSPAR	OLIVINE	PYROXENE	AMPHIBOLE	BLACK MICA	ALKALINE FELDSPAR	WHITE MICA	SILICA
2000	1	Anorthite	Forsterite	X	X	X	X	x	x
1000	2	Bytownite	Mgnesio-chrysolite	Enstatite	X	X	X	x	x
900	3	Labradorite	Chrysolite	Mnesio-hypersthene	Kupfferite	X	X	x	x
800	4	Andesine	Hyalosiderite	Hypersthene	Magnesio-Anthophyllite	Phlogopite	X	x	x
700	5	Oligoclase	Hortonolite	Ferro-hypersthene	Anthophyllite	Biotite	Orthoclase	x	x
600	6	Albite	Ferr–hortonolite	Eulite	Cummingtonite	Ferro-biotite	Perthite	Muscovite	x

500	7	Quartz	Fayalite	Ferrosilite	Grunerite	Lepidomelane	Albite	Paragonite	Quartz

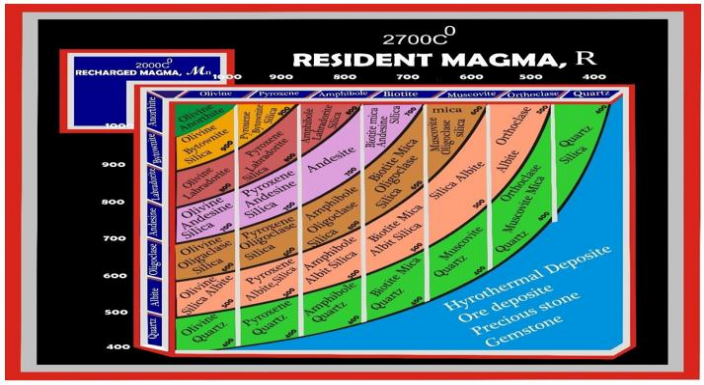

Figure 1. Model of Order of Crystallization of Minerals under thermodynamic change

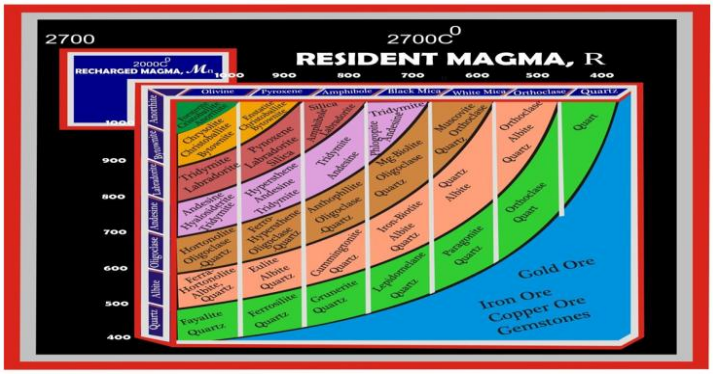

Figure 2. Model of Order of Crystallization of Minerals under thermodynamic change

Summary

Since the application of Power to Magma $\mathfrak{M}$, involves the use of Power series such as Euler, Taylor and Maclaurin series the following summaries were made using the following mathematical tools below.

1. $fn(x) = en$: Euler equation $(Z0)2$ $(Z0)3$ $(Z0)n$
2. Euler Series: $eZ0 = 1 + Z0 + \ + \ + \cdots \ldots \ldots . +$
2! 3! $n!$
3. $f(Z) = f0(0) + f1(0)Z + f11(0)(Z\ )2. . +. . f\eta(0)(Z\ )\eta{-}p$:Maclaurin's series
4. $f(x) = f0(a) + f1(a)(x - a) + f11\ (a)(x{-}\ a)2 + \cdots + f\eta\ (a)(x{-}\ a)\eta$: Taylor 2! $\eta!$ series

Conclusion

In conclusion, this research was focused on Taylor series $(f(x) = f0(a) + f1(a)(x - a) + f11\ (a)(x{-}\ a)2 + \cdots + f\eta\ (a)(x{-}\ a)\eta)$ and Maclaurin series in bridging the 2! $\eta!$ Gap between Bowen's and Goldschmidt concepts concerning the "problem of elemental substitution and distribution of chemical elements in rocks throughout the time of crystallization under thermodynamic change". With the context Euler series, Taylor series and Maclaurin series were helpful to predict and define all the minerals and rocks in a complex plane of magma $\mathfrak{M}$. In dynamic Magma $\mathfrak{M}$, the combined concepts of Bowen and Goldschmidt provide a concise

explanation of rock forming minerals, aiding students in mineralogy and petrology to comprehend rock evolution during magma crystallization. It utilizes mathematical methods such as Taylor series, and Euler series with the help of thermodynamic principles for the computation of numerical values of minerals into the complex plane and space.

Conflicts of Interest

The authors declare no conflict of interest

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